A MIXED INTEGER MODEL OF AN R&D PROJECT SELECTION/RESOURCE ALLOCATION PROBLEM

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A MIXED INTEGER MODEL OF AN R&D PROJECT SELECTION/RESOURCE ALLOCATION PROBLEM

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

This thesis presents a mixed integer programming model of Research and Development (R&D) project selection/resource allocation problem. R&D managers are periodically faced with the problem of allocating company resources, such as money and facilities, to several research projects or project areas. In addition, the manager must assign research personnel to the proposed projects.

The model presented in this thesis is intended to assist the manager with these tasks.

The first chapter describes the R&D environment and the project/selection resource allocation process. A general description is given which gives consideration to organizational structures and technological structures, and how the project/selection resource allocation process works within these structures.

The second chapter develops a framework for the model. The desirable capabilities and characteristics of any proposed model are listed. Areas of flexibility and limitation are discussed, weighing advantages and disadvantages.

Chapter III presents the proposed model of the thesis and the underlying assumptions are described and discussed. A solution algorithm is developed using Benders Partitioning Algorithm for Mixed Variables Programming

Problems [16].

Chapter V discusses some problem areas of implementation of the proposed model, going into more detail about the model's characteristics and capabilities.

Flexibilities and shortcomings which may arise during implementation are also discussed.

The last chapter lists conclusions and recommendations for further research into model design and areas of improvement upon the proposed model.

CHAPTER I

INTRODUCTION

Managers of research and development (R&D) face an annual problem of designing a portfolio of research projects which provide maximum benefit to the company and utilize the available resources efficiently. The uncertainties associated with identifying and defining the directions for project activity and predicting the benefits realized from the implementation of the results of a completed project are major causes of the difficulties involved with designing a portfolio of projects. The success of an R&D project is a function of many variables. Some of these variables the R&D manager can control, such as researcher assignments within the R&D organization and budgetary allocations which take place inside the R&D organization. Others the R&D manager cannot control, such as company personnel policies for the R&D organization and competitive forces from outside the R&D organization. In most R&D situations, project benefit cannot be measured in terms of dollars. Some other measure of return must be developed.

Several "benefit measurement" techniques [11] are available for the purpose of estimating project benefit. The benefit estimates can then be used to select a set of project proposals which result in maximum total benefit, while satisfying the resource constraints. Another approach considers different possible funding levels for each proposal resulting in an efficient allocation of resources. This requires the generation of a function which relates benefit measurements to all feasible funding levels.

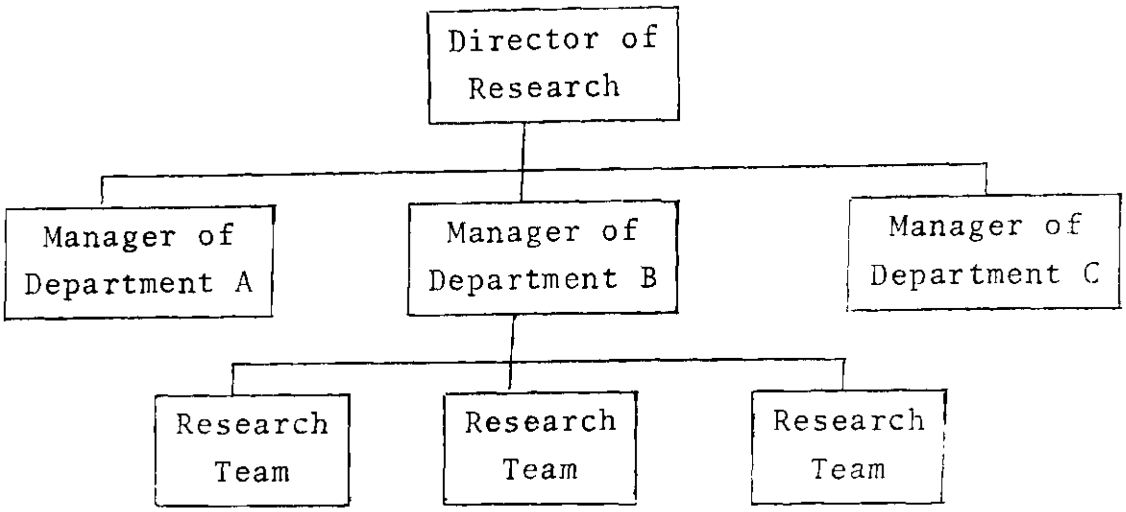
This thesis first presents a general description of the R&D project selection/resource allocation process based on several descriptive papers and model proposals. The paper then presents a project selection/resource allocation model. The mathematical formulation used is designed to maximize benefit by allocating available budget dollars to competing project proposals and assigning R&D manpower to the funded projects. The mathematical formulation has mixed variables, with the discrete variables being zero-one integers, representing researcher assignments. The continuous variables represent the project selection portion of the formulation. The optimizing solution algorithm used is Benders Partitioning Algorithm for mixed variables [16] which has proven to be an efficient algorithm for large scale programming applications [16,19,20,41,45] and is capable of more general applications then the Dantzig-Wolfe decomposition procedure [45]. The general description combined with the mathematical formulation results in a project selection/resource allocation model. Application of this model must be done on an individual basis due to the variety of sizes and structures of R&D organizations and of the types of projects that are involved. For example, a pharmaceutical company may have thousands of projects which consist of testing chemicals using standardized procedures [5,58] whereas a government organization may fund mission oriented, large scale projects which encompass several technical fields [44].

The remainder of this chapter will discuss the R&D environment with respect to the project selection/resource allocation problem. Chapter II discusses the capabilities required of a general project selection/resource allocation model, paying specific attention to desirable characteristics of models and types of portfolio analysis. Chapter III will present a mathematical formulation and a discussion of the assumptions involved with its application. A solution algorithm is then given to be used for selecting project proposals and allocating manpower using the formulation. Chapter IV discusses the implementation of the model through the entire project selection/resource allocation process. The flexibility of the formulation to perform the various types of portfolio analyses is discussed along with its capabilities to handle revisions of data values and modifications of the objective function and constraints. Finally, Chapter V evaluates the model's capabilities and recommendations of further development.

The R&D Environment

Literature which describes the R&D environment [2,3,4,7,9,18,20,28,32,33,35,59,61,63,74,75] and surveys other literature in the same area [2,10,23,39,51,52,67,68,70] is invaluable to anyone faced with the task of designing a mathematical model that can assist the R&D manager in solving the project selection/resource allocation problem. This chapter summarizes the descriptive material presented in the above references.

In [35], Freeland and Baker state that most business organizations are hierarchical in nature and that this can be verified by examining any company's organization chart. The R&D organization is no exception. A generalized description of the R&D hierarchical system is given in Figure 1(a) [35,55,65,66]. The R&D facility is frequently headed by a director of research. Under him may be several departments headed by managers and below each manager are the researchers, who are grouped into research teams by technical areas or project assignments. In [12], Baker, et al. have shown a dual hierarchical system composed of the organizational hierarchy described above and an activity hierarchy. The activity hierarchy was developed for the specific case presented in [12], but the general concept of a second hierarchy, based more on the technological organization of projects, can still be made (see Figure 1(b)). An illustrative second hierarchical structure consists of R&D



(b) TECHNOLOGICAL

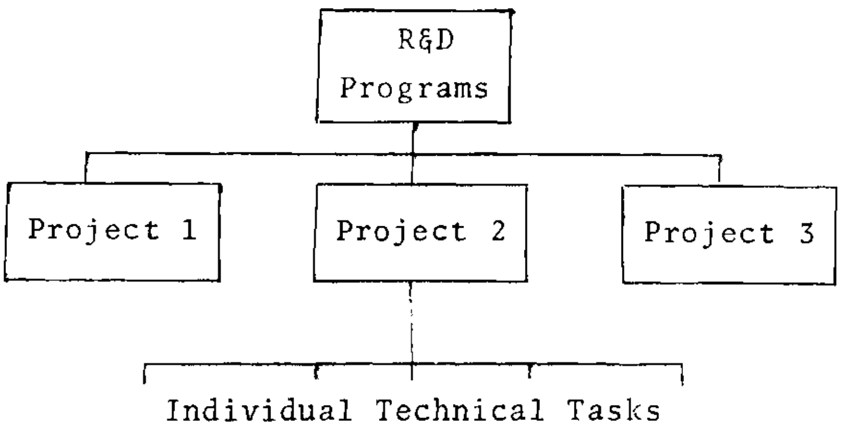


Figure 1. R&D Dual Hierarchical Structures

programs, which are composed of projects, which are composed of individual technical tasks. There can be a simple correspondence between these two hierarchies if researchers are assigned to individual technical goals, research teams are assigned to projects, and departments are assigned to R&D programs. However, the relationship becomes more complex in R&D organizations where departments are organized by company product or product line and are responsible for providing R&D support. Then each department may have responsibilities which involve several R&D programs, such as product improvement, process improvement, energy conservation, etc., and which also require a wide range of technical capabilities.

In Figure 2 a simplified diagram of the project selection/resource allocation process is presented. This diagram is basically condensed from the material in references [18,33,63,66]. The blocking of the process into eight phases is done primarily to organize subsequent discussion in this thesis, but is consistent with the descriptive literature.

In the first five phases, the project proposal data are collected and a portfolio is generated with resource allocations which correspond to project activity levels. (In the proposed model, the portfolio is generated by maximizing project benefit, though other methods are available.) The sixth through eighth phases involve the

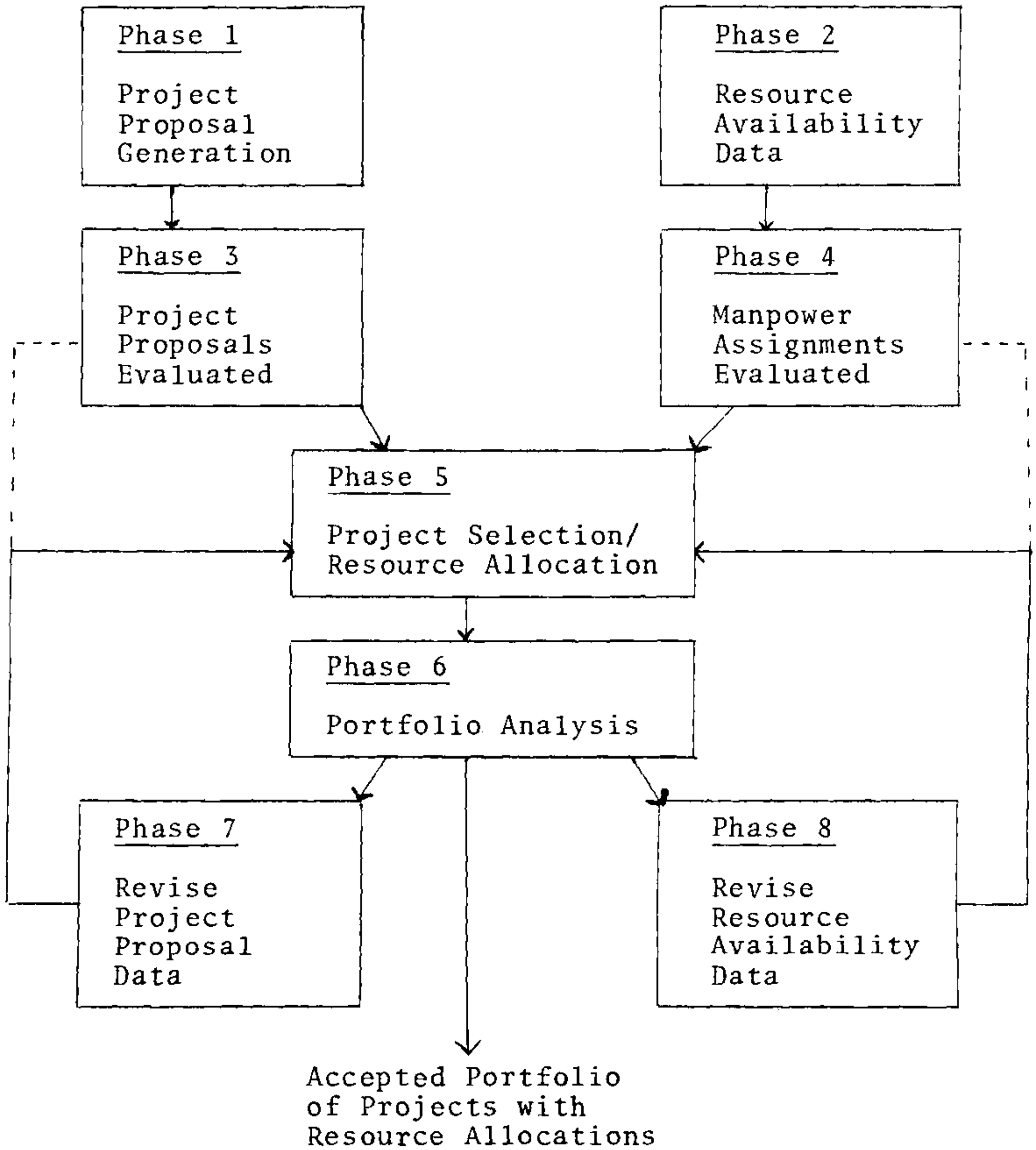


Figure 2. A Descriptive Model of the Project Selection/ Resource Allocation Process

analysis and refinement of the portfolio until an acceptable portfolio is produced.

The R&D Project Selection/Resource Allocation Process

Phase one represents the generation of R&D project proposals. These may be proposals for new projects or the continuation of current projects. Rubenstein [64] defines an idea as "a potential proposal for undertaking new technical work which will require the commitment of significant organization resources such as time, money, energy" and then defines a project proposal as an "idea" which has been submitted to an R&D manager for review. The data values associated with the project proposal state the requirements for "organization resources" and are in most cases generated at the manager and researcher levels of the organizational hierarchy. For the purpose of being able to compare project proposals, the data supplied are usually standardized by describing the project's objectives and resource requirements in common terminology and units of measure [8,11,28, 53,57].

The second phase shown in Figure 2 signifies the provision to the R&D manager of resource availability data, e.g. manpower limitations and budgetary guidance. This information is provided by the R&D manager's superordinate, the director of research or a company vice president, and may be defined in part by company accounting and personnel policies.

Phases three and four are concerned with evaluating project proposals and possible researcher assignments. Available techniques to perform these evaluations include economic analysis [1,6,25,27,29,30,50], decision theory [38,42,48,60], risk analysis [2,42,60], utility theory [27, 60], and other forms of benefit measurement [11].

Some of the benefit measurement techniques seem to be promising [8] since the data collection procedures can be designed with simplicity in conceptualization and computerization. Benefit measurement techniques are discussed in [11], and have been classified into three major categories in [8]. The first category is called the "comparative model" where one proposal is by some method compared to another proposal or a set of alternate proposals. The second category is the "scoring model" where criteria are selected and a score is assigned to each project, depending on how it meets each criterion. The third category is the "benefit contribution model" where a proposal's benefit is measured in terms of how closely it conforms to R&D program goals or requirements. All three of these categories require well-informed respondents to provide the subjective inputs by following a systematic procedure as prescribed by the technique used. The decision on which benefit measurement technique is used should be made with consideration towards data accessibility and ease of use.

Most models measure a project proposal's benefit after consideration only of the benefit that is accrued after a project is completed or implemented. Freeland [34] points out that utilization of resources, in particular manpower, is not accomplished efficiently when assignments are made based solely on the availability of researchers. Researcher performance can greatly effect a project's overall benefit and is determined by several factors, such as the researcher's technical capability, his desire to work on the project, the amount of time he has been assigned to the project, etc. Benefit is also accrued when project activity increases the R&D facility's capabilities. Indicators of this benefit include the increased experience of the researchers and the advancement of technological state-of-the-art. This benefit will be labeled as "research activity benefit," because it is accrued from project activities that occur within the R&D facility. It then follows to define "implementation benefit" as the benefit which is realized upon the completion and implementation of a project. The two benefits are estimated in phases three and four, respectively.

In phase five, the collected data estimates, including the project proposals, are used to generate a portfolio of selected projects and resource allocations. Methods available to generate portfolios [52] are multiple time period analysis [6,13,31,43,62], optimization analysis [5,24,25,36,56,76], simulation analysis [6,48] and scheduling analysis

[14,15,17,72]. Whichever method is used, it will be the heart of the model and should be capable of allowing the R&D manager to perform the types of portfolio analysis required in phase six.

In phase six, the R&D manager must refine and validate the portfolio of projects developed in phase five. Various types of analyses are required due to the amount of uncertainty that is inherent in the R&D process and which results in the use of a preponderance of subjective data. This analysis is also needed to overcome weaknesses or shortcomings of the model's representation of the R&D environment. The output of phase six is an acceptable portfolio of projects with resource allocations or a request to revise some project proposals including information that will provide guidance for the revisions, and/or a request to modify the availability of resources including information that will provide justification for the modifications. The provision of information for these "revisions" and "modifications" is an output of phase five that is essential to the decision making process. The information is generated, for the most part, by performing various "types of portfolio analyses."

In phases seven and eight, the "revisions" and "modifications" are made to the data values. These revisions and modifications may imply changes in the model's representation of the R&D project selection/resource allocation process or require the generation of additional data values for evaluating the revised project proposals and possible manpower assignments. The revised data values are entered into the process and the process recycles. The R&D manager will then proceed to iterate through the last four phases until he has completed the analysis to his satisfaction and has produced a final portfolio of project proposals with resource allocations.

This general description is unique in that the project selection process and resource allocation process are described as being done simultaneously. Nearly all the previous descriptions have described these processes as happening sequentially with the selection process being accomplished first. This description also shows the iterative process that takes place while a portfolio is being designed. Both Brandenburg [18] and Ruefli [66] have observed this iterative decision making process and have incorporated it into models of their own. Hierarchical considerations are also incorporated in the general description since phases one and seven can be performed at the researcher level, phases three through six can be performed at the department manager level, and phases two and eight can be performed at the director level (see Figure 1). Recent literature by Connolly [26] on diffuse decision making is also consistent with the description of an iterative decision making process operating within a hierarchical system. Connolly states that the organizational decision process cannot be pinpointed as a single event, but rather it is a process which encompasses several levels of organization and an extended period of time.

CHAPTER II

DESIRABLE CAPABILITIES AND CHARACTERISTICS OF A PROJECT SELECTION/RESOURCE ALLOCATION MODEL

Stipulation of the characteristics and capabilities which are desired in a project selection/resource allocation model is needed before it can be designed and implemented. This chapter discusses the characteristics which a useful R&D project selection/resource allocation model should have and what types of analysis an analyst should be able to perform using the model. The chapter then discusses why past efforts to model the project selection/resource allocation process have not been well received by R&D managers.

Desirable Characteristics of a Project Selection/Resource Allocation Model

Souder in [67,68] has listed many characteristics of R&D project selection/resource allocation models and classified them by five types of criteria (see Table 1). Souder claims that these groups of characteristics can be used to evaluate the usefulness of a project selection/resource allocation model.

The first of these classifications states that a realistic representation of the R&D environment is essential. If the model is predictive, data must be collected to give

Table 1. R&D Project Selection Models Performance Criteria and Their Characteristics*

1. <u>Realism Criterion Characteristics</u>

<u>Model includes</u>:
- Multiple objectives
- Multiple constraints
- Market risk parameter
- Technical risk parameter
- Manpower limits parameter
- Facility limits parameter
- Budget limits parameter
- Premises uncertainty parameter

2. <u>Capability Criterion Characteristics</u>

<u>Model performs</u>:
- Multiple time period analyses
- Optimization analyses
- Simulation analyses
- Scheduling analysis

3. <u>Flexibility Criterion Characteristics</u>

<u>Model applicable to</u>:
- Applied projects
- Basic projects
- Priority decisions
- Termination decisions
- Budget allocation applications
- Project funding applications

4. <u>Use Criterion Characteristics</u>

<u>Model is characterized by</u>:
- Familiar variables
- Discrete variables
- Computer not needed
- Special persons not needed
- Special interpretation not needed
- Low amount of data needed
- Easily obtainable data

5. <u>Cost Criterion Characteristics</u>

<u>Model has</u>:
- Low set-up costs
- Low personnel costs
- Low computer time
- Low data collection costs

*From Souder [68]

accurate estimates of project benefits. Parameters are needed to describe technical risks and resource limitations. The model then proceeds to design a portfolio which maximizes the total benefit from R&D activities. If the model is descriptive, as in the case of a simulation model, a thorough understanding of the R&D environment is needed. The model must then consider all the forces within this environment which affect the outcome of R&D activities. If the model is prescriptive, an understanding of the origins of forces which affect the success of R&D activities is needed to design policies which will control them.

The second classification is the group of characteristics which are concerned with the model's capability to perform various types of computational analyses needed to generate the project portfolios. The model may be capable of performing multiple time period analysis, optimization analysis, simulation analysis, or scheduling analysis. Multiple time period analysis is done when more than one budgetary cycle is considered during the decision making process. Optimization analysis is performed to generate project portfolios using a mathematical formulation which consists of two types of functions [7]:

(1) Objective functions which measure the total benefit contribution of a project portfolio, and

(2) constraint functions which represent the organizational environment within which the

decision must be made.

Techniques to find optimal or near optimal solutions are then applied to generate a project portfolio. Simulation analysis is where the model "imitates" the behavior of a system over a period of time [71]. Statistics are then gathered during the simulation and used to explore alternate strategies which will yield, in this case, a better portfolio of research projects with resource allocations. Scheduling analysis considers conflicts which may arise from use of common resources and priority or sequencing requirements of the proposed research activity.

The third classification of characteristics demonstrates the <u>flexibility</u> criterion which is needed. The model must be able to compare different types of projects, such as applied research and basic research projects. The model must also be flexible enough to handle priority and termination decisions. Budget allocation policies and project funding patterns can produce several acceptable portfolios and the model must be flexible enough to generate them for management analysis.

The <u>use criterion</u> pertains to the ease of use and the amount of time and manpower needed to perform the data collection and portfolio analyses. Using familiar terms and interpretations while describing the project selection/resource allocation process, enhances the manager's ability to understand the model and use its results for portfolio analysis and decision making.

Low cost is the fifth desirable characteristic. The costs of development, computerization, and data collection are all to be considered when designing a model.

Types of Portfolio Analyses

The discussion on phase six in Chapter I describes the "portfolio analysis" that follows phase five and is done primarily to refine and validate the portfolio which has been generated. Geoffrion and Graves [41] have listed the types of analyses that are used to properly deal with a system design problem. This list also applies to nearly any type of modeling exercise.

The types of portfolio analyses a model should provide for an analyst are as follows:

(1) Probationary exercises: Computer runs should be made to increase the credibility of the data and the model's realism. The analyst examines the data to make sure it is as complete and accurate as possible. He will also determine if the data will provide the model with a large number of feasible solutions so that the capability exists to perform the remaining types of portfolio analyses.

(2) Regional Optimization: The project proposals can usually be grouped by the R&D organization's structure consisting of departments and research teams, or grouped by technological areas. If the set of project proposals with

their possible manpower assignments can be grouped into independent subsets, regional optimization over each of these subsets can assist the manager and his subordinates with decisions on portfolio design. These preliminary decisions may include modifications of project proposals and the generation of an initial solution for global optimization analysis.

(3) Global Optimization: The department manager or director of the R&D facility must make decisions which cross organizational and technological boundaries. These decisions, for example, may allocate capital among departments or a researcher's time on different research teams. Objective functions or constraints are usually needed to describe these interrelationships which cross organizational and technological boundaries.

(4) 'What if ...?" questions: The analyst will want to consider possible variations in data, which may or may not be under the manager's control. He may want to look at the effects of hiring, firing, or transferring researchers. Forces from outside the R&D environment, such as those eminating from marketing, production, or sales groups, can present problems that are predictable as to "what" can happen, but not "when" or "if" they might occur. The analyst may then desire to design alternate strategies which can be implemented without having to reperform the portfolio generation phase.

(5) Sensitivity analysis: Since many of the data values are subjective estimates, the portfolio's sensitivity to error must be analyzed. This analysis is done to study the behavior of the portfolio as data values are varied over a range of values within the neighborhood of their original estimates.

(6) Continuity analysis: This analysis is similar to sensitivity analysis, but here the analyst is looking for discontinuities which might take place with small variations of data. A decision to transfer a small amount of capital from one technological area to another may cause major reassignments of personnel or changes in the composition of the project portfolio.

(7) Tradeoff analysis: The analyst may want to define and compare general policies. Examples would be: "What is the tradeoff point for maximizing benefit when considering allocation of resources to basic research versus applied research?" or "What type of technical diversification should researchers demonstrate? Should a researcher be given more projects related directly to his specific technical field or should he take part in projects which represent several fields of technology?".

(8) Priority analysis: The model should be able to indicate, when the decision is made to implement project proposals as active projects, which projects are most urgent. This output can assist the manager with the

scheduling of R&D activities.

Limitations Characteristic of Previous Models

Baker [7] points out that a very small portion of the normative models developed actually become implemented and reach the stage where managers express confidence in the model's output. In [7] is a summary of the "limitations inherent in the currently proposed normative models." These limitations are:

1. inadequare treatment of project and parameter interrelations with respect to both benefit contribution and to resource utilization.

2. inadequate treatment of uncertainty as it impacts on benefit measurement and parameter estimation.

3. inadequate treatment of multiple, interrelated decision criteria which have no common, natural underlying measure.

4. inadequate treatment of the time variant property of the parameters and criteria and the associated problem of continuity in the research program and staff.

5. a restricted view of the problem which (a) portrays a once-a-year investment decision rather than an intermittent stream of investment alternatives, (b) does not include such attributes as timing of the decision, generation of additional alternates, and recycling, (c) does not recognize the diversity of projects along the spectrum from basic research to engineering, and (d) views the problem as a decision event rather than as a hierarchical, diffuse decision process [26].

6. no explicit recognition and incorporation of the importance of individual R&D personnel.

7. the ability to establish and maintain balance in the R&D program; e.g. balance between basic and applied research, between offensive and defensive research, between breakthrough and improvement

orientations, between in-house and contracted projects, between product and process oriented projects, and between high risk/high payoff and low risk/low payoff projects.

Summary

The material presented in this chapter will be used in Chapter V to discuss and evaluate the proposed model. Souder's list of characteristics will in many cases not be complete since R&D organizations usually have characteristics which are peculiar to the environment in which they function. Some characteristics from each category are needed to proficiently perform the eight types of portfolio analysis. Realism characteristics are most important here because with each type of portfolio analysis, the analyst is validating the current portfolio by subjecting it to tests which simulate R&D activities and problems which can arise.

Baker's list of limitations points out areas where previous models have failed to adequately treat the problems of project selection and resource allocation. Most of these limitations can be related to a lack of one or more characteristics from Table 1. This list of limitations is used to compare the proposed model presented in this thesis with the previous

CHAPTER III

THE MATHEMATICAL FORMULATION AND SOLUTION ALGORITHM

This chapter starts with a mathematical formulation for the project selection/resource allocation stage of the R&D decision process presented in Figure 2. This is followed by a discussion of the assumptions associated with the structure of the formulation. Then Benders Partitioning Method for Mixed Variable Problems [16] is used to develop a solution algorithm.

The Mathematical Formulation

The formulation of the project selection/resource allocation model will be referred to as problem (P) and is written as follows:

Problem (P)

$$\text{MAX} \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} a_{ijk} X_{ijk} + \sum_{j=1}^{m} b_j Y_j \tag{1}$$

subject to

$$\sum_{i=1}^{n} \sum_{k=1}^{N} X_{ijk} \leq [M_j Y_j + E] \qquad j = 1,\ldots,m \tag{2}$$

$$\sum_{i=1}^{n} \sum_{k=1}^{N} X_{ijk} \geq [M_j Y_j - E] \qquad j = 1,\ldots,m \tag{3}$$

$$\sum_{j=1}^{m} c_j Y_j \leq B \tag{4}$$

$$\sum_{j=1}^{m} \sum_{k=1}^{N} X_{ijk} \leq N \qquad i = 1,\ldots,n \tag{5}$$

$$X_{ij(k+1)} - X_{ijk} \leq 0 \qquad \begin{array}{l} i = 1,\ldots,n \\ j = 1,\ldots,m \\ k = 1,\ldots,N-1 \end{array} \tag{6}$$

$$0 \leq Y_j \leq 1 \qquad j = 1,\ldots,m \tag{7}$$

$$X_{ijk} \text{ binary} \qquad \begin{array}{l} i = 1,\ldots,n \\ j = 1,\ldots,m \\ k = 1,\ldots,N-1 \end{array} \tag{8}$$

where

X_{ijk} represents the assignment, (if equal to one), of researcher i to project j for an additional k'th unit of time given that the researcher has already been assigned for (k-1) units of time to project j.

Y_j represents the activity level for project j which has a value between zero and one.

n is the number of researchers available.

m is the number of project proposals.

N is the number of discrete time increments into which the budgetary period is partitioned. The "k" subscript is used to indicate these increments. $(1/N)$'th of the budgetary period is defined as a "unit of time."

a_{ijk} is the estimated benefit of assigning researcher i to project j for the k'th additional unit of time, given that he has been assigned for (k-1) units of time to project j. These values are referred to as "research activity benefit."

b_j is the estimated benefit associated with project j when the project is undertaken with an activity level of $Y_j = 1$. These values are referred to as "implementation benefit."

c_j is the estimated cost in dollars associated with project j that is incurred when the project is undertaken with an activity level of $Y_j = 1$.

B is an upper limit on the total costs, the $c_j Y_j$ summed over all projects.

M_j is the estimated level of manpower that is needed to support an activity level of $Y_j = 1$ for project j. This value is expressed in the "units of time" as determined by the value of N.

E is defined as the acceptable absolute error of the product $M_j Y_j$ from the total manpower assignment determined by the X_{ijk} values, summed over the i and k indices. (Subsequent discussion in this chapter will give reasons why this parameter is required in the formulation.)

The objective function (1) of problem (P), maximizes the total benefit gained from the assignment of each researcher i and the completion of each project at an activity level Y_j. The portion of the objective function that represents the estimated benefit that will be accrued by making researcher assignments $\left(\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} a_{ijk} X_{ijk}\right)$ is a piecewise linear approximation to a nonlinear, concave function. Methods for expressing this function are discussed in Chapter IV.

The first set of constraints, equations (2) and (3), insure that the total researcher assignment time is within $\pm$ E of the estimated manpower requirement for each project, which is denoted by the product $M_j Y_j$. The analyst can make use of the value of E to improve the computational efficiency of the solution algorithm or to help perform sensitivity analysis (see Chapter IV).

Constraint (4) establishes the budgetary limit as specified by the value of B. The sum of the products $c_j Y_j$, for all projects, must not exceed this value.

Equations (5) state that for each researcher, no more than N units of time can be assigned for the defined budgetary period. Equations (6) specify the order in which each value of X_{ijk} is set equal to one. This is required since the value of a_{ijk} is the benefit from increasing researcher i's assignment time from (k-1) units of time to k units of time.

Constraints (7) define the range of values which the activity level Y_j can assume. The resultant value of Y_j can indicate to the R&D manager whether the project should be dropped, redesigned or accepted into the final portfolio. The value of Y_j operates as a "probability of success" value, as it is used to calculate expected values of benefit, cost, and manpower requirements. A more detailed discussion on the meaning of the values of Y_j will be given in Chapter IV. Constraints (8) require the researcher time to be assigned in discrete units of time equal to (1/N) times the budgetary period.

Underlying Assumptions

The following assumptions are required by the mathematical formulation and must be considered during implementation of the model. In Chapter IV it will be shown that some of these assumptions can be relaxed at the expense of additional costs and time associated with the operation of the model. The assumptions needed for problem (P) are:

1. The values of c_jY_j, b_jY_j, and M_jY_j vary linearly with the activity level, Y_j. Figure 3(a) shows the assumed

relationships between these products and Y. Figure 3(b) presents examples of what may be more realistic, j nonlinear relationships. Atkinson and Bobis in [6] present a model which uses nonlinear relationships for these estimates. It will be shown later in this chapter that a nonlinear relationship between M_jY_j and Y_j can be incorporated into the solution algorithm. Nonlinear relationships between c_jY_j, b_jY_j and Y_j are discussed in Chapter IV.

2. The partitioning of the budgetary period into N units of time indirectly makes the assumption that a researcher must be assigned to a project either zero time or at least one full unit of time. This feature is useful if the R&D manager assumes that it is unwise to assign a researcher to a project for less than (1/N)'th of the budgetary period or to assign a researcher to more than N projects.

3. The formulation assumes that a project's benefit is determined independently of the activity levels of other projects. This assumption is not always appropriate, e.g. some proposals may be parallel approaches to achieve similar goals, the starting of a project may be contingent on the completion of another project, or two projects must be completed before any benefits are realized.

4. The model assumes that researcher assignments can be made without considering the size or composition of the research team.

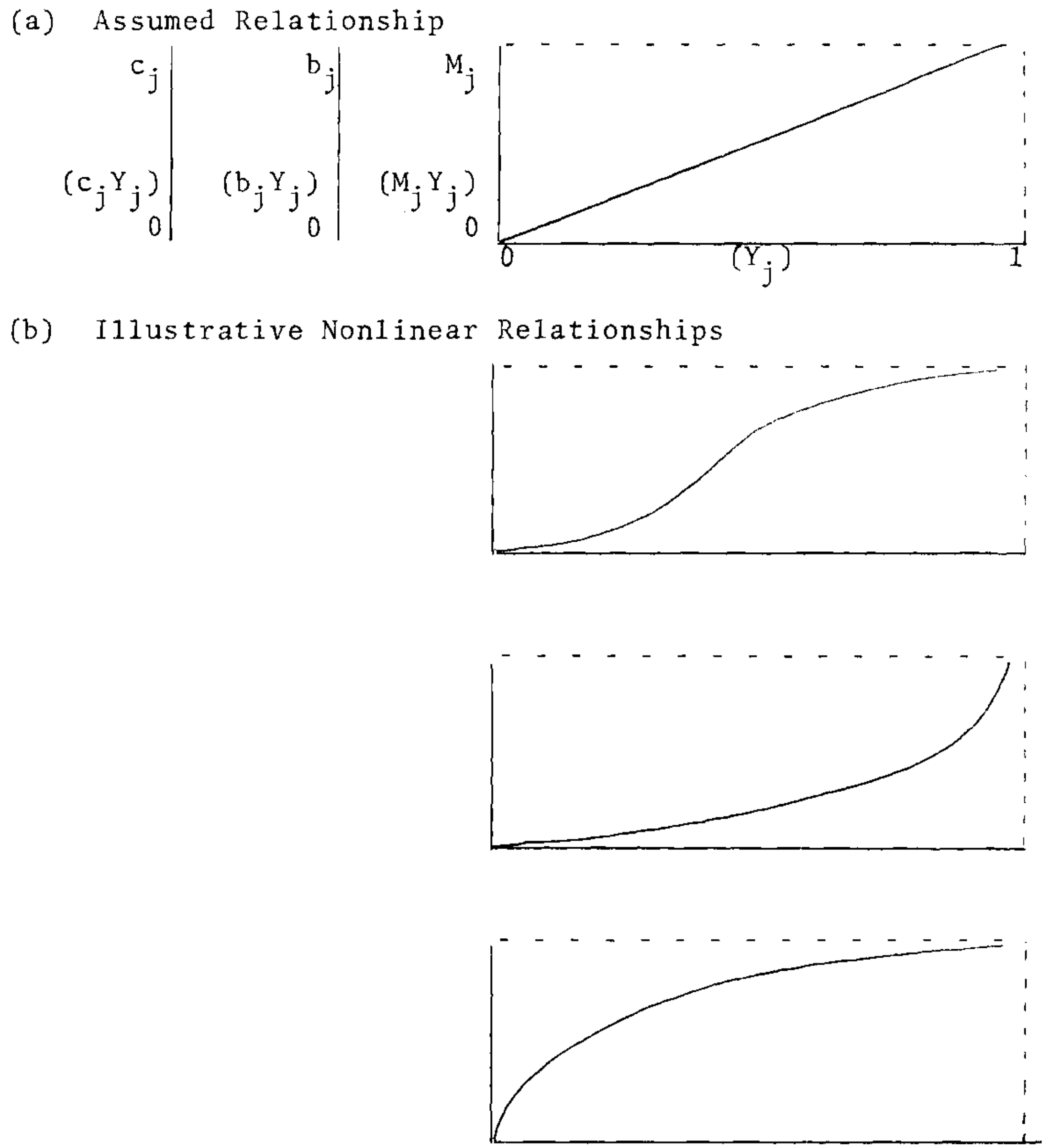


Figure 3. Relationships Between c_jY_j, b_jY_j, M_jY_j and the Activity Level Y_j

5. The formulation assumes that research activity benefit and implementation benefit can be expressed in the same units of measure and compared or added together.

6. The model assumes that during the budgetary period, data values will not vary from the original estimates.

7. The model assumes that decisions made during the previous budgetary period and for future periods are reflected in the benefit measurements.

8. The model assumes that benefit measurements will encompass risk measurements, uncertainties of project activity success and direction, comparison of various types of projects, and project timeliness or priority.

The Solution Algorithm

In order to facilitate the application of Benders Partitioning Method for Mixed Variables [16], problem (P) will be reformulated into a master problem and a subproblem. Assuming there is given a set of $\overline{X}_{ijk}$'s, which satisfy the constraints (5), (6), and (8), problem (P) reduces to the linear programming subproblem:

Problem (LP)

$$\text{MAX } Z_{-} = \sum_{j=1}^{m} b_j Y_j$$

subject to

$$-Y_j \le (1/M_j)(E - \sum_{i=1}^{n} \sum_{k=1}^{N} \overline{X}_{ijk}) \quad \forall j \quad (\alpha_j)$$

$$Y_j \le (1/M_j)(E + \sum_{i=1}^{n} \sum_{k=1}^{N} \overline{X}_{ijk}) \quad \forall j \quad (\beta_j)$$

$$\sum_{j=1}^{m} c_j Y_j \le B \quad \forall j \quad (\gamma)$$

$$0 \le Y_j \le 1 \quad \forall j \quad (\delta_j)$$

The dual of this problem is:

Problem (DLP)

$$\text{MIN } Z_{-} = \gamma B + \sum_{j=1}^{m} [\frac{E}{M_j}(\beta_j+\alpha_j)+\delta_j+ \frac{\beta_j-\alpha_j}{M_j} \sum_{i=1}^{n} \sum_{k=1}^{N} \overline{X}_{ijk}]$$

subject to

$$c_j\gamma + \delta_j + \beta_j - \alpha_j \ge b_j \quad \forall j$$

$$\alpha_j,\ \beta_j,\ \gamma,\ \delta_j \ge 0 \quad \forall j$$

Let S be defined as the set of feasible values for the dual variables in problem (DLP).

$$S = \{\alpha_j,\beta_j,\gamma,\delta_j \mid c_j\gamma+\delta_j+\beta_j-\alpha_j \ge b_j;\ \alpha_j,\beta_j,\gamma,\delta_j \ge 0;\ \forall j\}$$

Let P be defined as the set of extreme points of the set S.

$P = \{(\bar{\alpha}^p,\ \bar{\beta}^p, \bar{\gamma}^p,\ \bar{\delta}^p) \mid p$ is an extreme point of the set S.$\}$

where

$$\bar{\alpha}^p = \alpha_1^p, \alpha_2^p, \ldots, \alpha_m^p)$$

$$\bar{\beta}^p = \beta_1^p, \beta_2^p, \ldots, \beta_m^p)$$

$$\bar{\delta}^p = \delta_1^p, \delta_2^p, \ldots, \delta_m^p).$$

Let R be defined as the set of extreme rays of the set S.

$R = \{(\bar{\alpha}^r,\ \bar{\beta}^r, \bar{\gamma}^r,\ \bar{\delta}^r) \mid$ r is an extreme ray of the set S.$\}$

where

$$\bar{\alpha}^r = \alpha_1^r, \alpha_2^r, \ldots, \alpha_m^r)$$

$$\bar{\beta}^r = \beta_1^r, \beta_2^r, \ldots, \beta_m^r)$$

$$\bar{\delta}^r = \delta_1^r, \delta_2^r, \ldots, \delta_m^r).$$

Now problem (DLP) can be restated in terms of its extreme point solutions. (The set S is assumed bounded until proven otherwise, see Appendix A).

Problem (DLP')

$$\underset{p \in P}{\text{MIN}} \{\gamma^p B + \sum_{j=1}^{m} [\frac{E}{M_j} (\beta_j^p + \alpha_j^p) + \delta_j^p + \frac{\beta_j^p - \alpha_j^p}{M_j} \sum_{i=1}^{n} \sum_{k=1}^{N} \bar{X}_{ijk}]\}$$

Substituting this result into problem (P) and using objective function constraints gives the form of the master problem that will be used in the solution algorithm.

Problem (IP)

MAX Z_+

subject to

$$z_+ \leq \gamma^p B + \sum_{j=1}^{m} [\frac{E}{M_j}(\beta_j^p + \alpha_j^p) + \delta_j^p] + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} (a_{ijk} + \frac{\beta_j^p - \alpha_j^p}{M_j}) X_{ijk} \qquad \forall \ p \varepsilon P$$

$$\sum_{i=1}^{n} \sum_{k=1}^{N} X_{ijk} \leq N \qquad \forall i$$

$$X_{ij(k+1)} - X_{ijk} \leq 0 \qquad \forall i,j,k=1,\ldots,N-1$$

$$X_{ijk} \text{ binary} \qquad \forall i,j,k$$

If this solution to problem (DLP) is unbounded, then problem (LP) has no feasible solution, and therefore problem (P) has no feasible solution since the feasible region of problem (P) is a subset of the feasible region of problem (LP). For the unbounded solution to problem (DLP), the simplex method locates an extreme ray [37], which will be denoted by the extreme point ($\bar{\alpha}^p$, $\bar{\beta}^p$, γ^p, $\bar{\delta}^p$) and the extreme ray ($\bar{\alpha}^r$, $\bar{\beta}^r$, γ^r, $\bar{\delta}^r$). The value for the objective function

$$Z_- = \gamma B + \sum_{j=1}^{m} [\frac{E}{M_j} (\beta_j + \alpha_j) + \delta_j + \frac{\beta_j - \alpha_j}{M_j} \sum_{i=1}^{n} \sum_{k=1}^{N} \bar{X}_{ijk}]$$

approaches minus infinity along the half-line;

$$(\bar{\alpha}^p, \bar{\beta}^p, \bar{\gamma}^p, \bar{\delta}^p) + \lambda(\bar{\alpha}^r, \bar{\beta}^r, \bar{\gamma}^r, \bar{\delta}^r)$$

for $\lambda \geq 0$, scalar.

The inequality,

$$\gamma^r B + \sum_{j=1}^{m} [\frac{E}{M_j} (\beta_j^r + \alpha_j^r) + \delta_j^r + \frac{\beta_j^r - \alpha_j^r}{M_j} \sum_{i=1}^{n} \sum_{k=1}^{N} \bar{X}_{ijk}] < 0$$

is satisfied, so $\bar{X}$ does not satisfy the constraint,

$$\gamma^r B + \sum_{j=1}^{m} [\frac{E}{M_j} (\beta_j^r + \alpha_j^r) + \delta_j^r + \frac{\beta_j^r - \alpha_j^r}{M_j} \sum_{i=1}^{n} \sum_{k=1}^{N} X_{ijk}] \geq 0.$$

This constraint and the constraint,

$$Z_+ \leq \gamma^p B + \sum_{j=1}^{m} [\frac{E}{M_j} (\beta_j^p + \alpha_j^p) + \delta_j^p] + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} (a_{ijk} + \frac{\beta_j^p - \alpha_j^p}{M_j}) X_{ijk}$$

are added to problem (IP).

The general method of solution is to start with $\bar{X}_{ijk}$ values which satisfy the constraints (5), (6), and (8). These can be generated from the project proposals. Then solve problem (DLP) or (LP), whichever is easiest, and obtain the extreme point $(\bar{\alpha}^p, \bar{\beta}^p, \gamma^p, \bar{\delta}^p)$. Using this extreme point, an objective function constraint is added to problem (IP). If the solution to problem (DLP) is unbounded, an extreme ray $(\bar{\alpha}^r, \bar{\beta}^r, \gamma^r, \bar{\delta}^r)$ is found, and another constraint is added to problem (IP). Problem (IP) is then solved for new $\bar{X}_{ijk}$ values. The new $\bar{X}_{ijk}$ values were then entered into problem (DLP) and a second interation has begun. When the constraint generated by the extreme point cannot improve the previous solution to problem (IP), the algorithm is stopped.

If research assignments can be associated with independent sets of projects, grouped by research departments or technological areas, problem (LP) or (DLP) can be solved using decomposition or upper bounding methods [45]. Problem (IP) has the potential of being solved by an efficient version of Geoffrion's [40] Branch and Bound algorithm with backtracking. For each researcher-project assignment, where the coefficient a_{ijk} is zero, branching to these variables can be eliminated. Where X_{ijk} equals one, only the assignment variables X_{ijk} and $X_{ij(k+1)}$ are eligible for a change in value. Also a branching rule could be devised that will eliminate the feasibility test for the constraints (6). Additional efficiency can be gained if the constraints of problem (IP) can be reduced to a single "strongest surrogate constraint" [19,21,37] which has the property of eliminating many nonoptimal solutions.

The solution method suggested above solves problem (IP) once for every iteration. In [51] Unger develops a technique to solve a mixed-integer programming problem, where the integer programming problem is solved only once. Unger makes use of Geoffrion's [40] reformulation of Balas's implicit enumeration algorithm. Another similar method was developed by Lemke and Spielberg [46,47] in which a single search is also performed over the integer variables. The difference between the two algorithms is that Unger's algorithm finds the best completion of any given solution and then solves the linear programming problem, where Lemke and Spielberg's method searches for a solution to the integer problem having a value greater than the current lower bound, and then solves the corresponding linear programming problem.

Geoffrion and Graves [41] have found that relaxing constraints during the early iterations of the solution algorithm can speed up convergence significantly. If the initial value of $E = \text{MAX } M_j$ in the first iteration, then problem (LP) becomes a budgetary problem since the constraints associated with the value E become redundant to constraints (7). Another advantage of starting off with a large value for E, is that the initial set of researcher assignments may be excessive and cause problem (LP) to be infeasible if a sufficent number of Y_j's can not be driven to low values in order to meet the budgetary constraint (4). A rule for reducing E as the number of iterations increase can then be applied causing the constraints in problem (LP) and the generated constraints in problem (IP) to tighten, giving increasingly more accurate solutions at each iteration.

Varying the value for N can also increase computational efficiency if the additional calculations required to determine new values for a_{ijk} and M_j are not too excessive. The values of a_{ijk} and M_j are defined in terms of the unit of time measurement, which is (1/N)'th of the budgetary period. If N is changed between iterations, corrections must be made with the current values of the X_{ijk}'s to retain the total time of assignments of researchers to projects that have been made in previous iterations. During the initial few iterations, a small value for N generates only a few variables (X_{ijk}'s) for problem (IP). The solution algorithm will begin with "ballpark" estimates of researcher assignment times, which will become more accurate as N gets larger and partitions the budgetary period into smaller segments. Computational experience is needed with the formulation to develop rules for increasing the value of N and decreasing the value of E.

The solution algorithm requires an initial set of researcher assignments which satisfy the constraints (5), (6), and (8) of problem (P). These values can be obtained from recommended researcher assignments contained in the project proposals. These researcher assignment times must be expressed in terms of the "units of time" used in the formulation, then the appropriate X_{ijk}'s are set equal to one. If the manager has decided to begin the solution procedure with a small value for N, then an initial solution could be to find the best project (in terms of total benefit) for each researcher, and assign him to that project for the entire budgetary period. In either case, the researcher can not be assigned for a total amount of time which exceeds one budgetary period.

The solution algorithm to solve problem (P) will now be stated.

Step 0: Set $Z_+ = \infty$, $P = \emptyset$, and $R = \emptyset$. (An extreme point will be added to the set P at each iteration and an extreme ray will be added to the set R when it is needed.) Data values for n, m, b_j, c_j, M_j, and a_{ijk} for all indices have been determined in phases one through four of the model. An initial set of assignments, $\overline{X}_{ijk}$'s, which satisfy the constraints (5), (6), and (8), is derived from the initial set of project proposals. The values for E and N are set depending on how they will be varied, if at all, through iterations of the algorithm.

Step 1: Solve problem (LP) or (DLP), whichever is easiest to obtain an extreme point p, represented by the vector $(\overline{\alpha}^p, \overline{\beta}^p, \gamma^p, \overline{\delta}^p)$, and a selection of projects represented by their activity levels, $\overline{Y}_j \; \forall \; j$. If problem (DLP) is infeasible, then problem (P) is infeasible or unbounded. If problem (DLP) is unbounded, then an extreme ray, r, with an extreme point, p, is determined from the simplex tableau [37,p.34]. The extreme ray is added to the set R and the constraint

$$\gamma^r B + \sum_{j=1}^{m} [\frac{E}{M_j}(\beta_j^r + \alpha_j^r) + \delta_j^r] + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} (\frac{\beta_j^r - \alpha_j^r}{M_j}) X_{ijk} \geq 0$$

is added to problem (IP). Go to Step 3. If the solution to problem (DLP) is finite, continue to Step 2.

<u>Step 2</u>: (ε - optimality test)

$$z_+ \leq Z_- + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} a_{ijk} \overline{X}_{ijk} + \varepsilon$$

then a solution to problem (P) is

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} a_{ijk} \overline{X}_{ijk} + \sum_{j=1}^{m} b_j \overline{Y}_j .$$

The suggested value for ε is a percentage of the always feasible solution:

$$z_- + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} a_{ijk} \overline{X}_{ijk} .$$

<u>Step 3</u>: Add p to set P and the following constraint to problem (IP),

$$Z_+ \leq \gamma^p B + \sum_{j=1}^{m} [\frac{E}{M_j}(\beta_j^p+\alpha_j^p)+\delta_j^p] + \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{N} (a_{ijk} + \frac{\beta_j^p - \alpha_j^p}{M_j}) X_{ijk}.$$

Decrease E and increase N by predetermined rules, if these values are being varied.

Step 4: Solve problem (IP) giving new values for $\overline{X}_{ijk}$ $\forall$ i,j,k. Use the previous iteration's assignments (or the initial assignments) as a starting solution. A Branch and Bound method with backtracking is recommended. Utilization of surrogate constraints may be computationally practical. If problem (IP) is infeasible, then problem (P) is infeasible. Return to Step 1 to begin the next iteration.

A characteristic of the data values involved is the inaccuracy of their subjective estimates. Small changes in this data may be allowed, giving an improved solution, without effecting the accuracy of the project selection/resource allocation decisions. The key to perturbing data is the set S. If the set S is not affected by the changes in data, then the extreme points and rays will not have to be regenerated. Only the constraints generated by the extreme points and rays will have to be recalculated. Then reoptimization is performed by continuing with the solution algorithm and the current solution.

The data values found in the definition of S are the project proposal costs (c_j), the estimates of implementation benefits (b_j), and the number of project proposals (m). A method to handle changes in these values is discussed in the next chapter. The data values a_{ijk}, B, M_j, n, E, and N can all be perturbed within the solution algorithm, in Step 3, if the R&D manager or the analyst desires to interact with the solution procedure, or after a solution is obtained.

Some computational experiences with applications of Benders Partitioning Method [16,20,21,41,45,73] have been published and the algorithm has been found to converge to a solution quickly when using the "ε-optimality test." Reoptimization has also been found to be done quickly [41] when data values are perturbed during the solution procedure or after it was completed.

CHAPTER IV

SOME PROBLEM AREAS OF IMPLEMENTATION OF THE PROPOSED MODEL

This chapter discusses some of the problems which could arise during implementation of the proposed model. The first section provides a review of the general description of the decision making process, with discussion centering on the requirements associated with each of the eight phases. The second section discusses benefit measurement techniques which could be used to generate some of the benefit data needed in the mathematical formulation. The third section suggests some methods for performing portfolio analyses in phase six activities. The fourth section discusses each of the assumptions which follow from the mathematical formulation and when possible, suggests techniques to relax the assumptions.

The Model and the Decision Making Process

Phase one represents the generation of project proposals. They may come from ideas triggered by outside influences such as market demands, competitive forces, consumerism, etc. or within the R&D facility where someone has perceived a need or means [9] to apply a technology. As required by the model, these project proposals must contain the data needed to determine the values of the estimated manpower requirements (M_j), the estimated cost of performing the research (c_j), and the initial recommended researcher assignments (X_{ijk}) measured in the units of time as defined by the values of N and the length of the budgetary period.

The estimates, M_j and c_j, can contribute to the model's flexibility to generate alternative portfolios by the method in which they are determined from the project proposal data. The activity levels (Y_j) only range in value from zero to one. In terms of the decision making process, the value of Y_j could be used as an indicator to the analyst of whether the "proposed" project activity level, expressed by the estimated benefit, cost, and manpower data, should be revised upward or downward. For example, if the initial estimates of b_j, M_j, and c_j are viewed as associated with some recommended level of activity, say $y_j = 0.6$, then a final value of y_j substantially different than 0.6 would indicate that the estimates of b_j, M_j, and c_j should be considered for revision. These revisions are done in phases seven and eight and suggestions on how they can be done are given later in this section. Initially the values of M_j and c_j could represent the recommended resource requirements related to a $y_j<1$ or they could represent a maximum activity level related to $y_j = 1$ with M_j or c_j being set equal to an upper limit.

Phase two is where resource availability data values are determined. Sources for these data are the R&D manager's superordinates and company policies. The value of B is usually given as a budgetary guide and may be increased or decreased if justification to do so is developed in phase six. The number of available researchers (n) is also determined at this time.

In <u>phase three</u> the research activity benefit values are estimated. The approach of designing a "benefit function" which gives a continuous relationship between cumulative research activity benefit and cumulative researcher assignment time, can be efficient for generating values for the a_{ijk}'s. The next section in this chapter will discuss the generation of such functions. Whichever method is used to generate a benefit function, point estimates of research activity benefit will always be needed. Any of the types of benefit measurement techniques discussed in Chapter I can be used. The researcher appraisals, educational background, and working experiences should all be considered in estimating the research activity benefit values.

In <u>phase four</u>, the R&D manager, or his designate, determines the implementation benefits associated with each project proposal and then the value for b_j. The benefit data can be collected from R&D personnel, project clients, marketing personnel, technical forecasting groups, sales personnel, etc. The data should consider histories of similar projects, company and R&D objectives, competitive forces, timeliness of research results, etc. The analyst may need values of implementation benefit for a range of corresponding activity levels if he wants to determine if a nonlinear relationship exists between the product b_jY_j and the activity level Y_j. This relationship can be used during phase six analyses. The value given to b_j should correspond to the activity level, Y_j equal to one, as determined by the estimates of M_j and c_j.

In phase five, a solution for problem (P) can be found using the data generated in the earlier phases and the solution algorithm in Chapter III. This generates a portfolio of projects with resource allocations.

In phase six, the analyst determines the analyses needed to refine and validate the portfolio. If any of the assumptions discussed in Chapter III must be relaxed, additional analyses can be performed. Relaxing assumptions and performing portfolio analyses are discussed in later sections of this chapter.

In phases seven and eight, revisions and modifications to parameter estimates are made which are the result of previous portfolio analyses or are required to perform additional portfolio analyses. Revisions to the estimated values of b_j, c_j, and M_j are to be done using the activity level, Y_j, as an indicator of project effort. If Y_j is equal to a predetermined upper limit, then a simple method the analyst could use to calculate the new estimates, is to take the value of either c_j or M_j and increase it by a given percentage, then he increases the other estimate and b_j such that c_j, M_j and b_j describe a realistic project proposal. For example, c_j is increased to a new recommended funding level, c_j'. So that the new values of M_j and b_j reflect a realistic project proposal, a reduced value for the activity level is determined by the expression: $c_j'Y_j' = c_jY_j$ where Y_j equals one. Then the new manpower requirements (M_j') and implementation benefit (b_j') estimates are determined such that the products $M_j'Y_j'$ and $b_j'Y_j'$, along with $c_j'Y_j'$, describe a realistic project proposal with activity level Y_j'.

Generating Benefit Estimates

In Chapter I, three categories of benefit measurement techniques were briefly discussed [8]; the comparative models, the scoring models, and the benefit contribution models. Any of these techniques can generate point estimates of implementation benefit and research activity benefit. For the application of the proposed model, point estimates of implementation benefits for each project would be adequate, however, estimates are needed for each of the N values of a_{ijk} for each researcher-project combination.

Earlier in this chapter it was proposed that using a continuous cumulative benefit function to generate values for research activity benefit would be an efficient procedure. Alboosta and Holzman [2] used Burr's cumulative frequency function [22] to generate the probability of success curve for a project given a level of expenditure. They used this function because it has the following properties [2]:

1. The function is continuous of at least order one.

2. The function value is always between zero and one for all positive values of the independent variable.

3. The function's value is zero when the independent variable's value is zero.

4. The function's value approaches one as the independent variable goes to positive infinity.

5. The function is monotonically increasing.

6. The function has at most one point of inflection.

This same function may be used to represent the expected cumulative research activity benefit, given the cumulative researcher assignment time to a project. The mathematical expression, using Burr's function, to calculate cumulative benefit is:

$$a_{ij}(t) = [1-(1+t^{C})^{-K}]b_{ij}$$

where:

t is the fraction of the budgetary period which the researcher has been assigned to the project.

$a_{ij}(t)$ is the cumulative benefit accrued from assigning researcher i to project j for time t.

C is a positive constant which determines the shape of the curve.

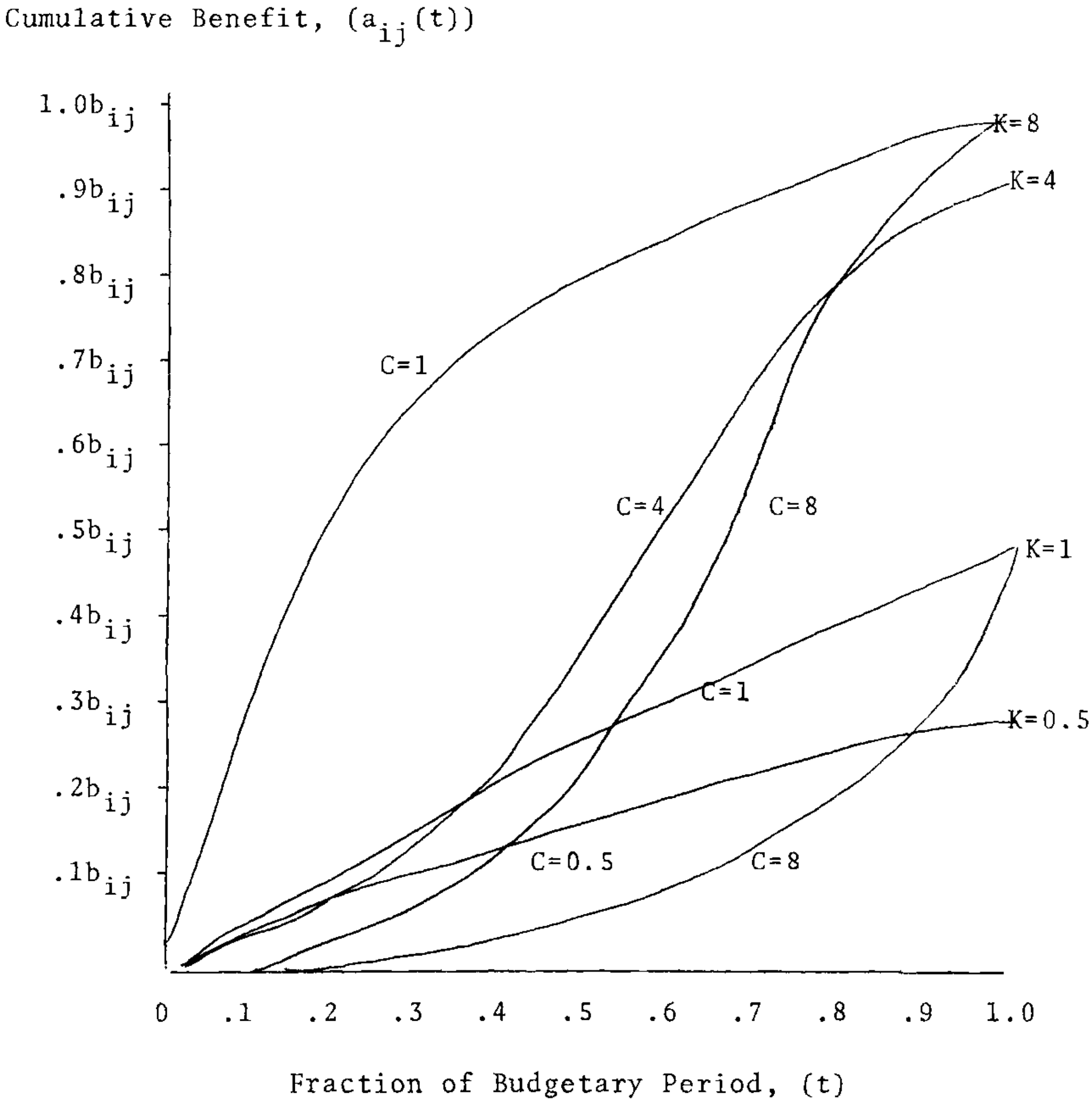
K is a positive constant which determines the value of the function at t = 1.

b_{ij} is the point estimate of maximum research activity benefit that could be accrued if researcher i is assigned to project j for the entire budgetary period.

The bracketed term of the function can be considered as a weighting factor that is a function of cumulative researcher effort. Figure 4 graphically shows the variety of curves which can be generated by only changing the two parameters C and K. The values of a_{ijk} can be calculated from the expression:

$$a_{ijk} = a_{ij}(k/N) - a_{ij}((k-1)/N) \qquad \forall\ k=1,\ldots,N$$

Other methods of generating curves of this type have been used by Baker, et al. [12], Souder [59] and Norden [55]. In the first two references, the curves were generated from answers to three of four questions which identify the end points, the shape, and the inflection points of the curve. Baker, et al. [12] asked three questions which determined the minimum, recommended, and maximum levels of funding for each project. A benefit estimate for each of the three funding levels was then determined. With this data, the



Benefit Function: $a_{ij}(t) = [1-(1+t^C)^{-K}]b_{ij}$

Figure 4. Burr's Cumulative Frequency Function

benefit function was determined to be either linear, concave, or convex and interpolation schemes were devised for each case to approximate the desired benefit values, given a proposed funding level.

Souder [69] used three "yes" or "no" questions to appraise each project (see Table 2). For each possible sequence of "yes" and "no" answers, Souder produced a general probability of success curve. A respondent's answers then determined which curve is used and the data from the fourth question fixed points on the selected curve.

Norden's [55] method was more analytical in that he used real data to plot "total effort" on a research project versus "total time" graph. These curves turned out to be "S" shaped, so Norden decided to use least squares regression analysis to fit a logistic growth function to the data. The form of the function was:

$$F(t) = \frac{K}{1 + \exp(a+bt+ct^2+dt^3)} .$$

Any one of these methods may be used to determine a benefit function and calculate the values for the a_{ijk}'s.

Performing Portfolio Analyses

The analyst may desire to perform a series of analyses to refine and validate the generated portfolio. Several types of portfolio analyses were presented in Chapter II.

Table 2. Souder's [69] Project Appraisal Questions

1. Do you expect to encounter primarily only familiar problems on this project? (yes or no)

2. Do you feel you have the basic technology needed to solve the kinds of problems which may arise during the life of this project? (yes or no)

3. Is the present technology capable of solving most of the problems that can occur on this project? (yes or no)

4. Can you reliably estimate the cost of each stage or phase of this project?

This section discusses how each type of analysis can be performed using the proposed model.

A probationary exercise should be performed before any other analysis begins. The initial set of proposed researcher assignments, the $\bar{X}_{ijk}$'s, can be checked to see if they satisfy constraints (5), (6), and (8). All estimated data values should be examined for accuracy.

Flexibility to generate several alternative funding patterns can be evaluated by examining the number of possible project assignments each researcher can have, i.e. those assignments represented by nonzero values of research activity benefit. If each researcher has only a few projects to which he can be assigned, the model is limited to the number of possible alternative portfolios it can produce.

Obviously, during the solution procedure, an infeasible solution for problem (P) requires an examination of the data values and the formulation.

Regional optimization can be performed by the model if the projects proposals are grouped by organization and/or technological structure. Strict independence of these groups is not necessary if global optimization is to follow. The model is applied only to one group of projects and selects a subset of these projects with resource allocations. Values for the budgetary and manpower limitations must be designated for each group. The results can be combined to form an initial solution for global optimization analysis.

Global optimization is where the model is applied to the entire set of project proposals. Interdependencies between the groups of projects used during regional optimization can be represented by adding constraints to the formulation or adding terms to the objective function. Examples of these techniques are presented in the next section. The allocation of money and manpower to the individual groups of project proposals used in the regional optimization analysis, will now be done analytically as part of the solution procedure for problem (P).

"What if...?" questions can be proposed by the R&D manager and used to demonstrate the flexibilities within the project portfolio. Two examples of questions which the manager can answer with the model are: "What alternate activity levels and researcher assignments should be made if a researcher is added or removed from the R&D staff?" "What alternate activity levels and researcher assignments should be made if a project proposal is added or deleted from the set of project proposals?" With the first question, an alternative portfolio can be found easily by adding or removing the data values associated with the (possibly hypothetical) researcher and reoptimizing the current solution. For the second question, the deleting of a project can be done by adding a constraint to problem (IP) which requires the manpower assignments to the project in question to be zero, (i.e., $\sum_{i=1}^{n} \sum_{k=1}^{N} X_{ijk} = 0$ for project j). The alternative portfolio is again found by reoptimizing the current solution. Adding a project involves adding more constraints to problem (DLP) which changes the set S. For this case an alternate portfolio is found by starting the solution procedure for problem (P) without the generated constraints.

Sensitivity and continuity analyses are discussed together here because the same procedures are used for each analysis. These analyses can be done by varying the data values of a_{ijk}, M_j, B, c_j, and b_j over a range of values within the neighborhood of their original estimates. An alternative portfolio is generated each time a data value changes. The alternate portfolios are examined for variations resulting from changes in the data values.

As discussed at the end of Chapter III, the values of a_{ijk}, M_j, and B can be perturbed without affecting the extreme points and rays of the set S. Determining the portfolio's sensitivity and continuity to changes in the value of c_j and b_j involves starting the solution algorithm without any generated constraints. This can prove costly if several values in the neighborhood of c_j and b_j are to be used. In addition, following similar procedures, the value of E can be varied upward from its lower limit to determine the portfolio's sensitivity to errors in the estimated value of M_jY_j.

Tradeoff analysis can be performed in the same manner as the "What if...?" questions are analyzed, but involve a series of questions which cover a range of strategies. Data generated from the series of portfolios is used to determine the best strategy. For the example given in Chapter II, "What is the tradeoff point for maximizing benefit when considering allocation of resources to basic research versus applied research?", the analyst can divide the set of proposals into two sets, one representing basic research projects and the other applied research projects. Then applying the model individually to each set (regional optimization), various strategies of assigning money and manpower to each group of projects are applied, and a curve representing the total benefit from both groups, for each strategy, is plotted. The strategy producing the maximum benefit would most likely be selected.

Priority analysis would be difficult to do unless the "urgency" or "timeliness" of a project proposal is reflected in its estimate of implementation benefit. The priorities can then be based on the estimate of implementation benefit for each of the accepted projects. Priority relationships can also be handled during global optimization analysis when they result from project interdependencies, which will be discussed in the following section.

Assumptions Revisited

The assumptions listed in Chapter III that follow from the mathematical formulation may not be entirely acceptable to the R&D manager. This section suggests methods which can be used to relax these assumptions.

<u>Assumption No. 1</u>: "The values of c_jY_j, b_jY_j, and M_jY_j vary linearly with the value Y_j." It was discussed at the end of Chapter III that a nonlinear function for the value of M_jY_j can be inserted into the solution algorithm by making corrections to the value of M_j at each iteration during Step 3.

To handle nonlinearities with the values of b_jY_j and c_jY_j, several solutions to problem (P) are needed. After each solution, the analyst examines the values of b_jY_j, c_jY_j and M_jY_j for each project. He then decides for which projects the values of benefit, cost and manpower requirements are unrealistic. If Y_j is not zero or one, the analyst then fixes the value of Y_j and one of the estimated values, (for example b_j), and determines new estimates for the other two values, (c_j' and M_j'), such that the three products, b_jY_j, $c_j'Y_j$, and $M_j'Y_j$, represent a realistic project proposal. If Y_j is zero, this indicates to the analyst that the proposal should be dropped. If Y_j is one, this indicates that a similar project of larger proportions should be designed and the new estimates of b_j, c_j, and M_j should be determined as discussed earlier in this chapter.

<u>Assumption No. 2</u>: "Researchers must be assigned to a project either zero or at least for (1/N)'th of the budgetary

period." As discussed in Chapter III, the value of N can be increased during the solution procedure. As N increases, the budgetary period is partitioned into smaller segments and therefore allowing the assignments to be made more efficiently. If the analyst decides that making researcher assignments efficiently is more important then setting a minimum assignment time or a maximum number of projects assigned to a researcher, he can let N continue to increase until a desired efficiency is obtained or computational costs become excessive.

Assumption No. 3: "Project benefits accrue independently of other R&D activities." Weingartner [76] has discussed different methods of accounting for project interdependencies in linear and integer programming formulations. His methods basically add constraints to the formulation, forcing relationships between projects to occur. An example of one relationship is where the activity level of one project proposal must be equal to the activity level of another project proposal. A constraint added to problem (IP) of the form:

$$(1/M_{(j1)}) \sum_{i=1}^{n} \sum_{k=1}^{N} X_{i(j1)k} = (1/M_{(j2)}) \sum_{i=1}^{n} \sum_{k=1}^{N} X_{i(j2)k}$$

will require approximately equal activity levels for projects (j1) and (j2).

Assumption No. 4: "Researcher assignments can be made without regards to the size or composition of the research team." If the R&D manager wants to designate an upper limit to the size of a research team, he may add to problem (IP) the constraints:

$$\sum_{i=1}^{n} X_{ij1} \leq U \qquad \forall\, j.$$

In this expression, U is the upper limit to the number of researchers that can be assigned to project j.

A second example where interdependencies exist with researcher assignments is a situation where researcher (i1) has an assistant, researcher (i2), and they both must be assigned to the same project, but not necessarily for equal amounts of time. A constraint added to problem (IP) of the form:

$$X_{(i1)j1} = X_{(i2)j1} \quad \forall\, j$$

will require a minimum assignment of researcher (i2)'s time to each project that researcher (i1) is assigned and vice versa. An expression added to the objective function of the form:

$$a_{(i1,i2)j} X_{(i1)j1} X_{(i2)j1}$$

can represent the additional benefit ($a_{(i1,i2)j}$) of assigning both research (i1) and (i2) to the same project.

Assumption No. 5: "Benefits eminating from project activity and implementation can be compared and added together." The argument that this statement is true and not an assumption at all, depends on which category of benefit measurement techniques are used to arrive at both point estimates of b_{ij} (see Figure 4), the maximum research activity benefit and b_j, the project implementation benefit.

Using a "comparative model," the analyst will end up with two sets of benefit measurements. Each set has a corresponding unit of measure, which are not related to each other through a common unit of measure. If one relationship can be found between research activity benefit and implementation benefit, then both sets of benefit measurements can be expressed in terms of the same unit of measurement.

Using a "scoring model" has more promise. The accepted technique is to score each project on how well it meets specific criteria, then weight and add these scores together. The researcher-project combinations would be scored according to how they meet their own specific criteria. An example of a research activity criterion is: "How technically qualified is researcher i to perform the tasks associated with project j?" A project implementation benefit criteria would include such project characteristics as how closely the project meets company objectives or the total impact of the project upon a project or manufacturing process. The criteria, for each benefit estimate b_{ij} and b_j, would then be assigned weights which reflect their relative importance. The weighted scores can then be added together to obtain total benefits.

"Benefit contribution models" would require estimations of how research activities contribute to company objectives or requirements. This would be difficult to do since for the most part, benefits from research activities are only realized within the R&D organization. Then, similar to the scoring model, weights must be assigned to each company objective, measuring their relative importance. The weighted benefit estimates can then be compared and added together.

Assumption No. 6: "Data values do not vary with time." This assumption cannot be relaxed during the project selection/ resource allocation procedure. The effects of small variations in estimated data can be examined by performing sensitivity and continuity analyses. Predictable problems, which may result from gross inaccuracies in the estimation of data, can be examined using "What if...?" question analysis. A good policy to follow would be to periodically update the estimated data values and regenerate an acceptable portfolio which does not vary unreasonably from the current portfolio. Operating the model for a shorter budgetary period can improve the decisions made. This is due to the increased accuracy of estimations which are made over a shorter period of time.

Assumption No. 7: "Decisions made during the previous budgetary period and for future periods, are reflected in the benefit measurements." In all likelihood, this assumption cannot be relaxed. Multiple time period analysis is required and a different type of formulation is needed with equations that describe relationships which connect one time period to the next.

Assumption No. 8: "Benefit measurements will encompass risk measurements, uncertainties of project activity success and direction, comparison of various types of projects, and project timeliness or priority." This assumption points out characteristics of the benefit measurement estimates which should be considered within their design. Inclusion of these characteristics can add validity to the resulting portfolio of projects.

Summary

This chapter has gone into more detail with some of the problems which can arise during the implementation of the proposed model. The project selection/resource allocation process was discussed in the context of what occurs during each phase of the decision making process. Suggested benefit measurement techniques are discussed including the use of benefit functions to generate continuous values of cumulative benefit for all feasible levels of effort. The third section discussed the types of portfolio analysis and how the mathematical formulation is used to perform them. The final section discussed each assumption that follows from the acceptance of the model and how they may be relaxed.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

A project selection/resource allocation model has been presented which consists of a general description of the decision making process and a mathematical formulation which is used to generate an optimal or near optimal portfolio of projects with resource allocations.

The general description recognizes the presence of dual hierarchical systems, both organizational and technical, and the diffused decision making process within the R&D environment. Unusual to previous models, project selection and manpower allocation are described as being performed simultaneously. The descriptions presented in the general model allows more adequate treatment of the project and parameter interrelations with respect to both benefit contribution and to resource utilization. The R&D manager, or analyst, becomes an integral part of the decision process by his performance of portfolio analyses, phase six activities.

The mathematical formulation, using individual values for benefits resulting from research activities and project implementation, determines the activity levels for projects, efficiently assigns manpower time, and allocates budget funds. The formulation uses Benders Partitioning Method to develop a mixed integer solution algorithm which performs the project selection and resource allocation analyses. Benders Partitioning Method has a computationally tested solution algorithm which has proven in other applications to be efficient for finding optimal or near optimal solutions and capable of handling variations in some data values. As a consequence, the formulation and solution algorithm can be used to perform several types of portfolio analyses, utilizing the analyst's experiences and abilities. This in turn gives the analyst the capability to analyze many problems which may occur and have impact on the portfolio design. The ability to perform "What if...?" question and tradeoff analyses account for a major portion of this capability.

Limitations with the model are:

1. The model is not designed to handle multiple time period analysis or scheduling of project activities.

2. Constraint and objective function modifications to problems (LP) and (DLP) modify the formulation to such an extent that it becomes impractical for computerization. As a result, project interdependencies must be handled by adding constraints and modifying the objective function of problem (IP).

3. A large number of variables can be generated in problem (IP), requiring the need to develop an efficient integer programming solution algorithm for problem (IP).

This also makes regional optimization analysis more desirable before global optimization is performed.

4. Much weight is placed upon the benefit measurements with respect to their treatment of multiple, interrelated decision criteria and the accuracy with which they represent these criteria. The assumption that the research activity benefits and the implementation benefits can be compared and added together, is essential to the model, but controversial to its validity.

Recommendations

The following is a list of recommendations for further research beyond the developments of this thesis.

1. Computational experience is needed to verify the described capabilities of the model for various types of R&D environments.

2. Various solution techniques for problems (LP), (DLP), and (IP) can be evaluated. Then their capabilities to incorporate various R&D environment characteristics could be examined.

3. The possibility of using a nonlinear programming formulation for problem (IP) and its advantages or disadvantages can be examined.

4. Possible extensions of this model's analysis are to include multiple time period considerations and scheduling of project activities.

5. More techniques could be developed to relax some of the assumptions associated with the mathematical formulation.

6. Other applications of mixed variable formulations in project selection/resource allocation models can be investigated because they could give added flexibility, capabilities, and realism to the modeling of the decision making process.

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